SUBJECT and GRADE	Mathematics Grade 11	
TERM 3	Week 3	
TOPIC	**Functions and Graphs: Finding the equation of functions**	
AIMS OF LESSON	• To find the equation of the Parabola, hyperbola and exponential functions if graph is given.	
RESOURCES	***Paper based resources***	***Digital resources***
	Please go to the Functions and Graphs section in your Mathematics Textbook.	Parabola: https://www.youtube.com/watch?v=5yecNfFyvF8 Hyperbola: https://www.youtube.com/watch?v=Mx9-3WqFV6c Exponential: https://www.youtube.com/watch?v=YYNYc6HP6sk https://www.youtube.com/watch?v=vmFiraM8qTw

INTRODUCTION

By now you should have dealt with the parabola, hyperbola and the exponential functions where you have sketched the functions and made deductions from the sketches. In this lesson we will determine the equations of the mentioned functions.

Recall: the general form for the,

- parabola: $y = ax^2 + bx + c$ and the turning point form: $y = a(x+p)^2 + q$, where $(-p;\ q)$ are the coordinates of the turning point. Note: when we determine the x-intercepts/ roots, we use: $y = a(x - x_1)(x - x_2)$ where x_1 and x_2 are the roots of the parabola.
- hyperbola: $y = \frac{a}{x+p} + q$ where $x = -p$ and $y = q$ are the equations of the asymptotes of the hyperbola.
- exponential function: $y = a.b^x + q$ where $y = q$ is the equation of the asymptote of the exponential function.

CONCEPTS AND SKILLS

LESSON 1: TO DETERMINE THE EQUATION OF A GIVEN PARABOLA

Example 1: Determine the equation of the given parabola.

Solution:

$y = a(x - x_1)(x - x_2)$

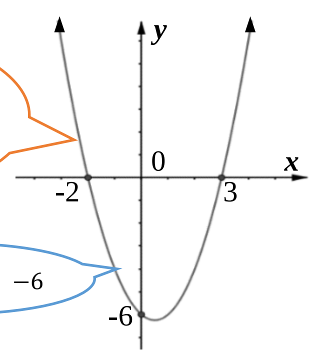
Substitute $\boldsymbol{x_1 = -2}$ and $\boldsymbol{x_2 = 3}$

$\therefore y = a[x - (-2)][x - 3)$

$\quad = a(x + 2)(x - 3) \ldots$ ①

2 Roots and **another point** are given: use

$y = a(x - x_1)(x - x_2)$



$x = 0;\ y = -6$

Now determine the value of a by substituting the point $\boldsymbol{(0;\ -6)}$ into ①: $y = a(x + 2)(x - 3)$

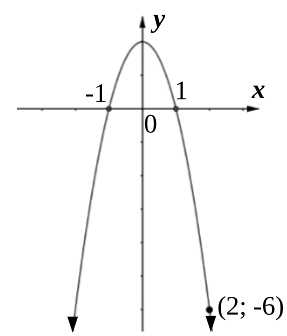
$\therefore\ \boldsymbol{-6} = a(\boldsymbol{0} + 2)(\boldsymbol{0} - 3)$

$\therefore\ -6 = a(-6) \Rightarrow a = 1$

$\therefore$ equation: $y = 1(x + 2)(x - 3) = x^2 - x - 6$

CAN YOU?

1. Determine the equation of the parabola given below.



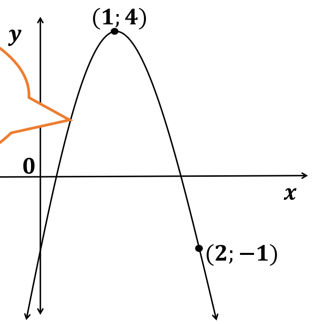
Answer: $\boldsymbol{y = -2x^2 + 2}$

Example 2: Determine the equation of the given parabola.

Solution:

Turning point and **another point** are given: use

$y = a(x+p)^2 + q$



$y = a(x+p)^2 + q$

Substitute $-p = 1 \Rightarrow \boldsymbol{p = -1}$ and $\boldsymbol{q = 4}$

$\therefore y = a(x-1)^2 + 4 \ldots ①$

Determine the value of a by substituting the point $(\mathbf{2}; \mathbf{-1})$

into ①: $y = a(x-1)^2 + 4$

$\therefore \mathbf{-1} = a[(\mathbf{2}) - 1]^2 + 4$

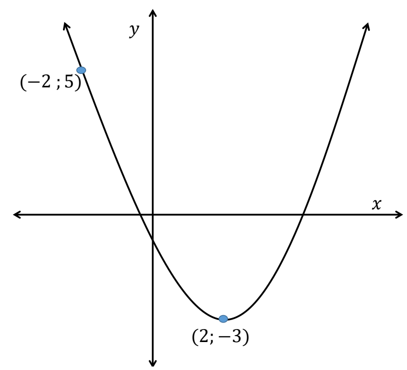
$\therefore -1 = a[1]^2 + 4$

$\therefore -1 - 4 = \boldsymbol{a} = \mathbf{-5}$

$\therefore$ equation: $y = -5(x-1)^2 + 4$ OR $y = -5x^2 + 10x - 1$

Take note of the different forms of the equivalent equation

Determine the equation of the parabola given below in the form, $y = a(x+p)^2 + q$.



Answer: $y = \frac{1}{2}(x-2)^2 - 3$

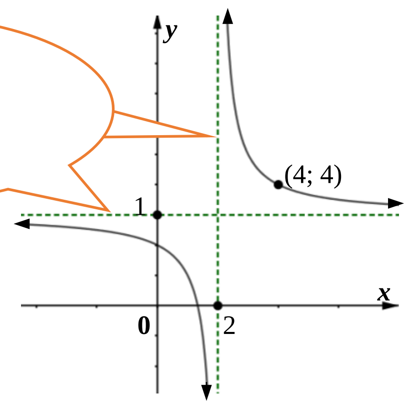
Do Exercises form your Textbook: Finding equation of Parabola

LESSON 2: TO DETERMINE THE EQUATION OF A GIVEN HYPERBOLA

Example 3: Determine the equation of the hyperbola in the diagram:

Solution:

The equations of the asymptotes are,
$x = 2$ and $y = 1$



$y = \frac{a}{x+p} + q$

Substitute $-p = 2 \Rightarrow \boldsymbol{p = -2}$ and $\boldsymbol{q = 1}$

$\therefore y = \frac{a}{x-2} + \mathbf{1} \quad \ldots ①$

Determine the value of $\boldsymbol{a}$ by substituting the point $(\mathbf{4}; \mathbf{4})$

into① : $y = \frac{a}{x-2} + \mathbf{1}$

$\therefore \mathbf{4} = \frac{a}{\mathbf{4}-2} + 1$

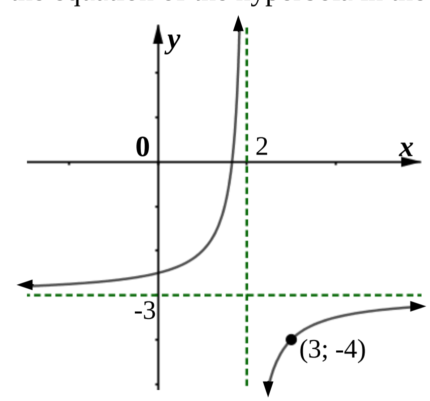
$\therefore 4 = \frac{a}{2} + 1$

$\therefore 3 = \frac{a}{2}$

$\therefore 6 = a \quad \Rightarrow$ Equation: $y = \frac{6}{x-2} + 1$

CAN YOU?

Determine the equation of the hyperbola in the diagram:



Answer: $y = \frac{-1}{x-2} - 3$

Do Exercises from your Textbook: Finding equation of hyperbola

Western Cape Government
Education

Directorate: Curriculum FET

LESSON 3: TO DETERMINE THE EQUATION OF AN EXPONENTIAL FUNCTION

Example 4: Determine the equation of the exponential function below:

Solution:

We must find the values of ***a*, *b*** and ***q***

$y = a.b^x + q$

Substitute $\boldsymbol{q} = \mathbf{4}$

$\therefore y = a.b^x + 4$ ...①

Substitute the y-intercept (0; 2) into ① to determine ***a***

$\therefore \mathbf{2} = a.b^{\mathbf{0}} + 4$

$\therefore 2 = a.1 + 4$

$\therefore -2 = a \Rightarrow y = -2.b^x + 4$ ...②

Substitute the point (1; -2) into ② to determine ***b***

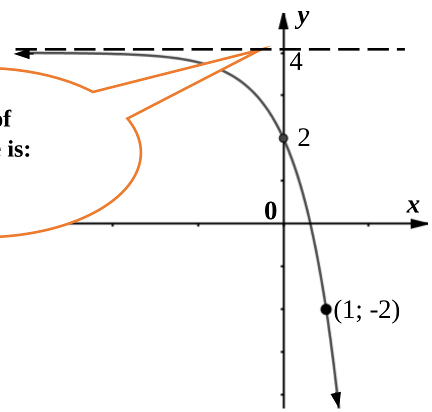
$\therefore \mathbf{-2} = -2.b^{\mathbf{1}} + 4$

$\therefore -6 = -2.b$

$\therefore 3 = b$

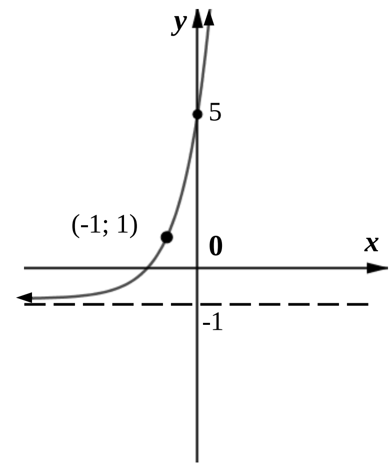
Equation: $y = -2.3^x + 4$

Equation of Asymptote is:
***y* = 4**



CAN YOU?

Determine the equation of the following exponential function



Revise exponential laws

Answer: $y = 6.3^x - 1$ **OR** $y = 2.3^{x+1} - 1$

ACTIVITIES/ASSESSMENT	Mind Action Series	Platinum	Classroom Mathematics	Everything Mathematics
	Ch 4 Pg: 63/64; 66/67; 71/73 and 78-80	Topic 5 Pg. 90-91; 98-99; 104-105; 106-109; 114-115	Ch 5 Pg: 109-112; 118-124; 125-131	Ch 5 Pg: 161-163; 181-183 and 191-196

CONSOLIDATION

Parabola:

- *If roots and a point are given; use* $y = a(x - x_1)(x - x_2)$ *where* x_1 *and* x_2 *are the roots; substitute other point to determine a.*
- *If turning point and another point are given; use* $y = a(x + p)^2 + q$ *where* $(-p;\ q)$ *are the coordinates of the turning point; substitute other point to determine a.*

Hyperbola:

- *Use* $y = \frac{a}{x+p} + q$ *where* $x = -p$ *and* $y = q$ *are the equations of the asymptotes; substitute other point to determine a.*

Exponential function:

- *Use* $y = a.b^x + q$ *where* $y = q$ *is the equation of the asymptote; substitute y-intercept to determine a and another point to determine b (if not given).*